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Göttingen, April 07, 2018

MAGKS doctoral course "Political Economics", spring 2018

Problem Set I

Please hand in your solutions in the lecture on Fri 20.04.2018, or send a pdf to the above email-address. It is not necessary to type solutions; hand-written solutions are acceptable as long as they are legible. Groups of up to three participants may co-operate.

Problem 1

Society is composed of an odd number $I > 0$ of families, labeled $i = 1, 2, ..., I$, each consisting of a parent who votes and a child. Family i is characterised by its income $y^i > 0$ and the child's ability $a^i > 0$. Average income and average ability are y and a, respectively. The utility of family i is

$$u^i = u(c^i, z^i) = c^i + z^i \,,$$

where c^i is the family's consumption and z^i is human capital of the family's child.

Human capital is produced by combining ability and per capita spending on public education, denoted by $s \geq 0$, according to

$$z^i = f(s) \cdot a^i \,.$$

The production function $f(s)$ satisfies $f'(s) > 0, f''(s) < 0$ for all $s > 0$, and $f'(0) > 1/a > f'(y)$, $f'(0) > y^i/(ya^i) > f'(y)$ for all $i = 1, 2, ..., I$.

(a) Derive the education level s^* which maximises the sum of utilities (utilitarian welfare maximum).

Spending on public education is financed by a proportional income tax at rate τ, $0 \leq \tau \leq 1$.

(b) State the indirect utility function of family i. Are preferences single-peaked? Derive the education level s^i most preferred by family $i = 1, 2, ..., I$.

(c) Assume that all children have the same ability, $a^i = a$ for $i = 1, 2, ..., I$. Which education level s_y is the Condorcet winner? When is $s_y \gtreqless s^*$? Interpret this result and discuss which case is most likely to hold empirically.

(d) Assume that all families have the same income, $y^i = y$ for $i = 1, 2, ..., I$. Which education level s_a is the Condorcet winner? When is $s_a \gtreqless s^*$? Interpret this result. Do you see difficulties in finding out the empirically relevant case?

(e) Characterise the Condorcet winner s_{ya} for the general case where both ability and income vary across families. When is $s_{ya} \gtreqless s^*$?

(f) Assume that ability of child i is given by

$$a^i = \gamma y^i + \epsilon^i \,,$$

with $\gamma > 0$ and a random component ϵ^i. Random components ϵ^i are independent of incomes and of each other and satisfy $E(\epsilon^i) = 0$ for all $i = 1, 2, ..., I$. Discuss whether and why ability could be linked to income in such a way. Assume that the vote takes place before random components are realised, and compare the Condorcet winner to the education level which maximises expected utilitarian welfare.

Problem 2

For some one-dimensional policy $q \in [0, \overline{q}]$, let citizen i's indirect utility be given by

$$W(q; \alpha^i) = \left(q - \frac{1}{\alpha^i} \right)^2 ,$$

where α^i is a preference parameter satisfying $\alpha^i \geq 1/\overline{q}$.

(a) Draw a graph of the indirect utility function for two different individuals i and i'. Are these preferences single-peaked?

(b) Show that the preferences satisfy the single crossing condition (Persson/Tabellini 2002, Def. 3, p. 23). Which policy is the Condorcet winner?

Problem 3

There are two periods $1, 2$. A country decides on the per capita amount of public investment g which has to be paid for in period 1 and is used in period 2.

Individual i's private consumption in periods 1 and 2 is $c_1^i \geq 0$ and $c_2^i \geq 0$. His or her preferences are represented by the utility function

$$u^i(c_1^i, c_2^i, g) = c_1^i + \delta^i \left(c_2^i + \ln g \right) .$$

The discount factors δ^i are distributed over the interval $[\underline{\delta}, \overline{\delta}]$, where $0 < \underline{\delta} < 1 < \overline{\delta}$. The median is denoted by δ^m. All individuals i have identical income $y^i = y > \overline{\delta}$ in each period.

To finance public investment, the government may levy lump sum taxes t_1 and t_2 in periods 1 and 2, respectively. Taxes may differ across periods but must be identical across individuals. Moreover, the government may issue public debt d per capita in period 1 which has to be repaid in period 2, with an interest rate of 0. Fiscal policy instruments are restricted to satisfy $0 \leq t_1, t_2 \leq y$ and $0 \leq d \leq g$.

(a) State individual i's indirect utility as a function of public investment g and public debt d. Why does a median voter result hold here (no proof required)? Explain intuitively why the median preferred policy is the equilibrium in a Downsian model of competing office motivated politicians.

(b) Solve individual i's optimisation problem to find his or her most preferred policy (g^i, d^i). Why is $d^i = 0$ or $d^i = g^i$ for most i? Explain how (g^i, d^i) depends on the individual discount factor δ^i. State the median preferred outcome (g^m, d^m).

The country agrees to a fiscal compact which outlaws public debt.

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(c) How does the democratically chosen policy change compared to the outcome (g^m, d^m) from (b)?

(d) Who loses, who wins from the fiscal compact? Explain. Would a majority of voters support the fiscal compact?

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